A coalgebraic approach to supervisory control of partially observed Mealy automata

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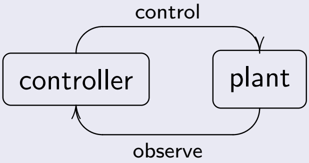
Outline

1. Introduction
2. Supervisory control (not using coalgebra)
3. Coalgebraic formulation
4. Solution to the problem
5. Conclusion

Introduction

Control problem

Given a $\boxed{\text{plant}}$ and a $\boxed{\text{spec}}$, design a $\boxed{\text{controller}}$ such that

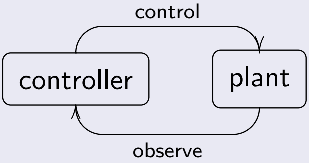


satisfies $\boxed{\text{spec}}$.

Introduction

Control problem

Given a plant and a spec, design a controller such that



satisfies spec.

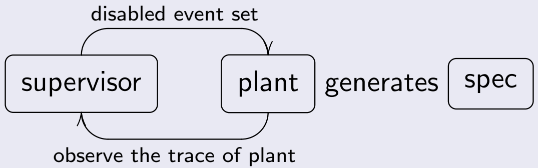
Our interest

- **When does a controller exist?**
- How do we design the controller?

Supervisory control

Control theory for discrete event systems [Ramadge and Wonham 1987]

communication networks, manufacturing systems, traffic systems



plant — deterministic partial automaton (X, A, δ, x_0)

spec — non-empty prefix closed language over A

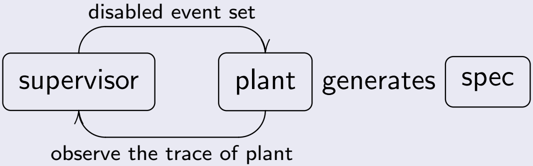
supervisor — function from a trace to a disabled event set

$S : A^* \to \mathcal{P}(A)$

Settings

- **Uncontrollable event** [Ramadge and Wonham 1987]
  event set $A = A_c + A_{uc}$, supervisor $S : A^* \to \mathcal{P}(A_c)$
  - A_c:controllable event set
  - A_{uc}:uncontrollable event set (not disabled by a supervisor)
- **Partial observation** [Ramadge and Wonham 1988, Cieslak et.al 1988]
  event set $A = A_o + A_{uo}$, supervisor $S : (A_o)^* \to \mathcal{P}(A_c)$
  - A_o:observable event set
  - A_{uo}:unobservable event set (not observed by a supervisor)
- **Partially observed Mealy automata** [Takai and Ushio 2009]
  plant modeled by a Mealy automaton
  supervisor $S : (B_o)^* \to \mathcal{P}(A_c)$
  - input event:$A = A_c + A_u$
  - output event:$B = B_o + B_u$

Our approach



plant	$\mathcal{M} \xrightarrow{m} (1 + B \times \mathcal{M})^A$	partial Mealy automaton
spec	$\mathcal{L} \xrightarrow{l} (1 + \mathcal{L})^A$	partial automaton
supervisor	$\mathcal{S} \xrightarrow{\langle o,t \rangle} \mathcal{P}(A_c) \times \mathcal{S}^{B_o}$	Moore automaton

Plant, Spec and Supervisor, coalgebraically

- **Plant** : $\mathcal{M} \xrightarrow{m} (1 + B \times \mathcal{M})^A$

$$\mathcal{M} = \left\{ M : A^* \rightharpoonup B^* \;\middle|\; \begin{array}{c} M \text{ is prefix- and length-preserving.} \\ \mathrm{dom}(M) \neq \emptyset. \end{array} \right\}$$

$m(M)(a) = \text{if } a \in \mathrm{dom}(M) \text{ then } \langle M(a), M_a \rangle \text{ else } \perp.$

where $M_a(w) = \mathsf{tail} \circ M(aw)$.

- **Spec** : $\mathcal{L} \xrightarrow{l} (1 + \mathcal{L})^A$

$\mathcal{L} = \{L \subseteq A^* \mid L \text{ is prefix-closed and nonempty.}\}$

$l(L)(a) = \text{if } a \in L \text{ then } L_a \text{ else } \perp.$

where $L_a := \{w \in A^* \mid aw \in L\}$.

- **Supervisor** : $\mathcal{S} \xrightarrow{\langle o,t \rangle} \mathcal{P}(A_c) \times \mathcal{S}^{B_o}$

$\mathcal{S} = \{S : (B_o)^* \to \mathcal{P}(A_c).\}$

$o(S) = S(\varepsilon),\; t(S)(b) = S_b,$ where $S_b(w) = S(bw)$.

Coinductive definition of supervisory composition

$$
\begin{tikzcd}
\mathcal{S} \times \mathcal{M} \arrow[r, dashed, "\exists!\ /"] \arrow[d, "\mathsf{spv}"'] & \mathcal{L} \arrow[d, "l", "\mathsf{final}"'] \\
(1 + \mathcal{S} \times \mathcal{M})^A \arrow[r, dashed, "(\mathrm{id}_1 + /)^A"'] & (1 + \mathcal{L})^A
\end{tikzcd}
$$

$$
\begin{aligned}
\mathcal{S} &= \{S : (B_o)^* \to \mathcal{P}(A_c)\} \\
\mathcal{M} &= \{M : A^* \rightharpoonup B^* \mid \cdots\} \\
\mathcal{L} &= \{L \subseteq A^* \mid \cdots\}
\end{aligned}
$$

$$
\mathsf{spv}\,\langle S, M\rangle\,(a) =
\begin{cases}
\langle S_b, M_a\rangle & \text{if } M \xrightarrow{a|b} M_a \wedge a \notin o(S) \wedge b \in B_o, \\
\langle S, M_a\rangle & \text{if } M \xrightarrow{a|b} M_a \wedge a \notin o(S) \wedge b \in B_u, \\
\perp & \text{otherwise.}
\end{cases}
$$

$/ : \mathcal{S} \times \mathcal{M} \to \mathcal{L}$ is the supervisory composition.
S/M represents a language generated by the controlled plant.

Formulation of supervisory control problem

Supervisory control problem

Given a plant $M \in \mathcal{M}$ and a specification $K \in \mathcal{L}$, find a supervisor $S \in \mathcal{S}$ satisfying

$$S/M = K.$$

$/ : \mathcal{S} \times \mathcal{M} \to \mathcal{L}$

- $\mathcal{S} = \{S : (B_o)^* \to \mathcal{P}(A_c).\}$
- $\mathcal{M} = \left\{ M : A^* \rightharpoonup B^* \;\middle|\; \begin{array}{c} M \text{ is prefix- and length-preserving} \\ \mathrm{dom}(M) \neq \emptyset. \end{array} \right\}$
- $\mathcal{L} = \{L \subseteq A^* \mid L \text{ is prefix-closed and non-empty.}\}$

Comparison

- Supervised product [Komenda & van Schuppen 2005]

$$(M/N)_a =$$

$$\begin{cases} M_a/N_a & \text{if } M \xrightarrow{a} \wedge N \xrightarrow{a}, \\ \left(\bigcup_{\langle M', M\rangle \in \text{Aux}} M'_a\right)/N_a & \text{if } M \not\xrightarrow{a} \wedge \exists M' \in DK : M' \approx M \text{ s.t. } M' \xrightarrow{a} \wedge N \xrightarrow{a} \wedge a \in A_c \cup A_o, \\ 0/N_a & \text{if } (\forall M' \in DK : M' \approx M) M' \not\xrightarrow{a} \wedge N \xrightarrow{a} \wedge a \in (A_{uc} \cap A_o), \\ M/N_a & \text{if } M \not\xrightarrow{a} \wedge N \xrightarrow{a} \wedge a \in A_{uc} \cap A_{uo}, \\ \emptyset & \text{otherwise.} \end{cases}$$

- Our work

$$\text{spv}\,\langle S, M\rangle\,(a) =$$

$$\begin{cases} \langle S_b, M_a\rangle & \text{if } M \xrightarrow{a|b} M_a \wedge a \notin o(S) \wedge b \in B_o, \\ \langle S, M_a\rangle & \text{if } M \xrightarrow{a|b} M_a \wedge a \notin o(S) \wedge b \in B_u, \\ \perp & \text{otherwise.} \end{cases}$$

$$S(w) = A_c \setminus \{a \in A_c \mid \exists u \in A^* : (K_0 \xrightarrow{ua}) \wedge (P \circ M_0(u) = w)\}$$

Partial bisimulation relation

Definition

Let (X, ξ) and (Y, η) be $(1 + -)^A$-coalgebras.
A partial bisimulation relation is a binary relation $R \subseteq X \times Y$ satisfying (1), (2), and (3).

(1)similarity $\forall a \in A$, $\forall x, x' \in X$, $y \in Y$, $\exists y' \in Y$,

$$x \; R \; y \wedge x \xrightarrow{a} x' \implies y \xrightarrow{a} y' \wedge x' \; R \; y'.$$

(2)controllability $\forall a \in A_u$, $\forall x \in X$, $\forall y, y' \in Y$, $\exists x' \in X$,

$$x \; R \; y \wedge y \xrightarrow{a} y' \implies x \xrightarrow{a} x' \wedge x' \; R \; y'.$$

(3)observability $\forall a \in A_c$, $\forall x \in X$, $\forall y, y' \in Y$, $\exists x' \in X$,

$$x \; R \; y \wedge y \xrightarrow{a} y' \wedge (\exists q \in X, \; (x \approx q) \wedge (q \xrightarrow{a})) \implies x \xrightarrow{a} x' \wedge x' \; R \; y'.$$

$$\approx = \left\{ \langle x, x' \rangle \;\middle|\; \exists w, w' \in A^*, x_0 \xrightarrow{w} x, x_0 \xrightarrow{w'} x', P \circ M(w) = P \circ M(w'). \right\}$$

When does a supervisor exist?

Theorem

Given a plant $M_0 \in \mathcal{M}$ and a specification $K_0 \in \mathcal{L}$, the following two conditions are equivalent.
(1) $\exists S \in \mathcal{S}$, $S/M_0 = K_0$
(2) There exists a partial bisimuration relation $R \subseteq \mathcal{L} \times \mathcal{L}$ *such that* $K_0 \; R \; \mathrm{dom}(M_0)$.

When does a supervisor exist?

Theorem

Given a plant $M_0 \in \mathcal{M}$ and a specification $K_0 \in \mathcal{L}$, the following two conditions are equivalent.
(1) $\exists S \in \mathcal{S},\ S/M_0 = K_0$
(2) There exists a partial bisimuration relation $R \subseteq \mathcal{L} \times \mathcal{L}$ such that $K_0\ R\ \mathrm{dom}(M_0)$.

$(2) \implies (1)$

$$S(w) = A_c \setminus \{a \in A_c \mid \exists u \in A^* : (K_0 \xrightarrow{ua}) \wedge (P \circ M_0(u) = w)\}$$

is a desired supervisor.

Modified normality

Problem

When no supervisor satisfies the specification, find the largest sublanguage of the specification.

- In general, there doesn't exist the largest controllable and observable sublanguage. (not closed under the arbitrary union)
- Therefore, we introduce a notion of *modified normality*. (closed under the arbitrary union)

Modified normality

Problem

When no supervisor satisfies the specification, find the largest sublanguage of the specification.

- In general, there doesn't exist the largest controllable and observable sublanguage. (not closed under the arbitrary union)
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Compute the largest controllable and modified normal sublanguage of the specification.

Controllable and modified normal relation

Definition

Let (X, ξ) and (Y, η) be $(1 + -)^A$-coalgebras.
A *controllable and modified normal relation* is a binary relation $R \subseteq X \times Y$ satisfying (1), (2), and (3).

(1)similarity $\forall a \in A$, $\forall x, x' \in X$, $\forall y \in Y$, $\exists y' \in Y$,

$$x \; R \; y \wedge x \xrightarrow{a} x' \implies y \xrightarrow{a} y' \wedge x' \; R \; y'$$

(2)controllability $\forall a \in A_u$, $\forall x \in X$, $\forall y, y' \in Y$, $\exists x' \in X$,

$$x \; R \; y \wedge y \xrightarrow{a} y' \implies x \xrightarrow{a} x' \wedge x' \; R \; y'$$

(3)modified normality $\forall a \in A_c$, $\forall x \in X$, $\forall y, y' \in Y$, $\exists x' \in X$,

$$x \; R \; y \wedge y \xrightarrow{a} y' \wedge (\exists q \in X, \exists a' \in A, (x \approx q) \wedge (q \xrightarrow{a'})) \implies x \xrightarrow{a} x' \wedge x' \; R \; y'.$$

Properties of modified normality

- controllable and modified normal relation
  $\implies$ partial bisimulation relation
- Let $\{K_i\}_{i \in I}$ be a family of prefix-closed languages.
  $\forall i \in I$, $\exists$controllable and modified normal relation R_i such that $K_i \ R_i \ L$
  $\implies$ $\exists$controllable and modified normal relation R such that $\left(\bigcup_{i \in I} K_i\right) R \ L$.

Supremal controllable and modified normal

Theorem

Let K and L be two prefix closed languages and $\tilde{R}_0$ be the greatest fixpoint of Φ_{R_0}.
$\exists K' \in \mathcal{L}$ such that $K' \subseteq K$ and $\exists$ controllable and modified normal relation R such that $K'\ R\ L$.
$\implies K\ \tilde{R}_0\ L$ and beh $\langle K, L\rangle$ *is the supremal controllable and modified normal sublanguage.*

$$\Phi_{R_0} : \mathcal{P}(R_0) \to \mathcal{P}(R_0),\ R_0 = \{\langle K_w, L_w\rangle \mid w \in K \cap L\}$$

$$\Phi_{R_0}(H) = \left\{ \langle x, y\rangle \in H \;\middle|\; \begin{array}{c} \forall a \in A_u : y \xrightarrow{a} y' \implies x \xrightarrow{a} x' \wedge x'\ H\ y' \text{ and} \\ \forall a \in A_c : y \xrightarrow{a} y' \wedge (\exists q \in X, (q \to) \wedge x \approx_M^{x_0} q) \\ \implies x \xrightarrow{a} x' \wedge x'\ H\ y'. \end{array} \right\}$$

Conclusion

Summary

- Coalgebraic formulation of the supervisory control problem
- Necessary and sufficient condition for the existence of a supervisor
- Algorithm to compute the largest controllable modified normal sublanguage

Future work includes:

- Categorical characterisation of partial bisimulations
- (Non)linear system and hybrid system